Homework 7

**Problem 1**

A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p(x,y)** | | **y** | | |
| 0 | 1 | 2 |
| **x** | 0 | .10 | .04 | .02 |
| 1 | .08 | .20 | .06 |
| 2 | .06 | .14 | .30 |

1. What is P(X = 1 and Y = 1)?

P(X = 1 and Y = 1) = 0.10

1. What is P(X<=1 and Y<= 1)?

P(X<=1 and Y<= 1) = 0.10 + 0.04 + 0.08 + 0.20 = 0.42

1. Give a word description of the event {X != 0 and Y!= 0} and compute the probability

of this event.

{X != 0 and Y!= 0} => {X = 1 or X = 2} and {Y = 1 or Y = 2}

P(X != 0 and Y!= 0) = 0.20 + 0.06 + 0.14 + 0.30 = 0.7

1. Compute the marginal pmf of X and of Y. Using Px(X)  what is P(X<=1)?

Px(0) = P(X = 0 and Y = 0)  + P(X = 0 and Y = 1) + P(X = 0 and Y = 2) =

= 0.10 + 0.04 + 0.02 = 0.16

Px(1) = P(X = 1 and Y = 0)  + P(X = 1 and Y = 1) + P(X = 1 and Y = 2) =

= 0.08 + 0.20 + 0.06 = 0.34

Px(2) = P(X = 2 and Y = 0)  + P(X = 2 and Y = 1) + P(X = 2 and Y = 2) =

= 0.06 + 0.14 + 0.30 = 0.5

Py(0) = P(X = 0 and Y = 0)  + P(X = 1 and Y = 0) + P(X = 2 and Y = 0) =

= 0.10 + 0.08 + 0.06 = 0.24

Py(1) = P(X = 0 and Y = 1)  + P(X = 1 and Y = 1) + P(X = 2 and Y = 1) =

= 0.04 + 0.20 + 0.14 = 0.38

Py(2) = P(X = 0 and Y = 2)  + P(X = 1 and Y = 2) + P(X = 2 and Y = 2) =

= 0.02 + 0.06 + 0.30 = 0.38

P(X <= 1) = P(X = 0 or X = 1) = P(X = 0) + P(X = 1) = 0.16 + 0.34 = 0.5

1. Are X and Y independent rvs? Explain.

If X and Y are independent => P(X = 0 and Y = 0) = P(X = 0) × P(Y = 0)

P(X = 0 and Y = 0) = 0.10

P(X = 0) × P(Y = 0) = 0.16 × 0.24 = 0.0384

0.10 ≠ 0.0384 => X and Y are not independent

**Problem 2**

Suppose 51% of the individuals in a certain population have brown eyes, 32% have blue eyes, and the remainder have green eyes. Consider a random sample of 10 people from this population.

P(brown) = 0.51

P(blue) = 0.32 C(n, k) =

P(green) = 0.17

1. What is the probability that 5 of the 10 people have brown eyes, 3 of 10 have blue eyes, and the other 2 have green eyes?

P = C(10, 5) × (0.51)5 × C(5, 3) × (0.32)3 × C(2, 2) × (0.17)2 = 0.082

1. What is the probability that exactly one person in the sample has blue eyes and exactly one has green eyes?

P = C(10, 1) × (0.32)1 × C(9, 1) × (0.17)1 × C(8, 8) × (0.51)8 = 0.022

1. What is the probability that at least 7 of the 10 people have brown eyes? [Hint: Think of brown as a success and all other eye colors as failures.]

P = C(10, 7) × (0.51)7 × (0.49)3 + C(10, 8) × (0.51)8 × (0.49)2 + C(10, 9) × (0.51)9 × (0.49)

+ C(10, 10) × (0.51)10 = 0.189

**Problem 3**

Two components of a computer have the following joint pdf for their useful lifetimes X and Y:

f(x, y) = xe-x(1+y) , x0, y0

1. What is the probability that the lifetime X of the first component exceeds 3?

P(X > 3) = dxdy = 0.05

dy = dy = dy = ) =

dx = - = (-) – (-) = = 0.05

1. What are the marginal pdfs of X and Y? Are the two lifetimes independent? Explain

fX(x) = dy = dy = dy = ) =

fY(y) = dx = dx(1+y) = d =

( + d = (- =

X and Y are independent if

1. fXY(x, y) = fX(x) × fY(y) is true

fXY(x, y) ≠ × => X and Y are not independent

1. What is the probability that the lifetime of at least one component exceeds 3?

P(X3 or Y 3) = 1 - P(X3 and Y 3) = 1 - dxdy =

= 1 – + + – 1 = 0.3

dy = dy = dy = ) = - +

dx = -dx + dx = - = - - + 1

**Problem 4**

An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X = the number of points earned on the first part and Y = the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the accompanying table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p(x, y) | | y | | | |
| 0 | 5 | 10 | 15 |
| x | 0 | .02 | .06 | .02 | .10 |
| 5 | .04 | .15 | .20 | .10 |
| 10 | .01 | .15 | .14 | .01 |

1. If the score recorded in the grade book is the total number of points earned on the two parts, what is the expected recorded score E(X + Y)?

E(X + Y) = E(X) + E(Y) = 5.55 + 8.55 = 14.1

|  |  |  |  |
| --- | --- | --- | --- |
| x | 0 | 5 | 10 |
| P(x) | 0.2 | 0.49 | 0.31 |

E(X) = 0 × 0.2 + 5 × 0.29 + 10 × 0.31 = 5.55

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| y | 0 | 5 | 10 | 15 |
| P(y) | 0.07 | 0.36 | 0.36 | 0.21 |

E(Y) = 0 × 0.07 + 5 × 0.36 + 10 × 0.36 + 15 × 0.21 = 8.55

**Problem 5**

A shipping company handles containers in three different sizes: (1) 27 ft3 (3\*3\*3), (2) 125 ft3 , and (3) 512 ft3 . Let Xi (i = 1, 2, 3) denote the number of type i containers shipped during a given week. With i = E(Xi) and 2i = Var(Xi), suppose that the mean values and standard deviations are as follows:

μ 1 = 200 μ2 = 250 μ3 = 100

σ1 = 10 σ2 = 12 σ3 = 8

1. Assuming that X1, X2, X3 are independent, calculate the expected value and standard deviation of the total volume shipped. [Hint: Volume = 27X1 + 125X2 + 512X3.]

E(Volume) = E(27X1 + 125X2 + 512X3) = 27 × E(X1) + 125 × E(X2) + 512 × E(X3) =

27 × 200 + 125 × 250 + 512 × 100 = 87850

Var(Volume) = Var(27X1 + 125X2 + 512X3) = 272 × Var(X1) + 1252 × Var(X2) +

5122 × Var(X3) = 272 × 100 + 1252 × 144 + 5122 × 64 = 19100116

Std = = 4370

1. Would your calculations necessarily be correct if the Xis were not independent? Explain.

if the Xi’s not independent => E(X1 + X2 + X3) = E(X1) + E(X2) + E(X3)

Var(X1 + X2 + X3) = Var(X1) + Var(X2) + Var(X3) +

2Cov(X1, X2) + 2Cov(X2, X3) + 2Cov(X1, X3)

The calculations are incorrect if Xis not independent.

**Problem 6**

A pizza place has two phones. On each phone the waiting time until the first call is exponentially distributed with mean one minute. Each phone is not influenced by the other. Let X be the shorter of the two waiting times and let Y be the longer. It can be shown that the joint pdf of X and Y is

f(x, y) = 2e-(x+y) , 0 < x < y <

1. Determine the marginal density of X

fX(x) = dy = 2dy = -2 = 2

1. Determine the conditional density of Y given X = x

fY/X(y/x) = = =

1. Determine the probability that Y is greater than 2, given that X = 1.

P(Y > 2 / X = 1) = dy = - = =

1. Determine the conditional mean of Y given X = x.

E(Y/ X = x) = dy = dy = dy = -d

= - - (-dy = x - = x + 1

1. Determine the conditional variance of Y given X = x.

E(Y2/ X = x) = dy = dy = dy =

-d= - + dy = x2 - 2 d = x2 –

- + dy = x2 + 2x - = x2 + 2x + 2

Var(Y) = E(Y2/ X = x) – [E(Y/ X = x)]2 = x2 + 2x + 2 – (x + 1)2 = 1